Valid Models Only LLC.
BME 350: Introduction to BME Design
1620 BBBB

MATLAB (30 points)- Due March 10th, 2026 at 11:59PM
Submit written answers and figures together in a **single document**, submitted as a **pdf file**. Submit **code in a single .m file** with sections for each problem. You should have only two files included in your submission!

Course learning objectives developed by this MATLAB Assignment:

1) Create meaningful figures in MATLAB that communicate findings from V&V testing
2) Identify how Finite Element Analysis (FEA) applies to Verification & Validation (V&V) testing of biomedical applications

Specific assignment objectives:
1. Refresh or learn basic MATLAB code to plot solutions to differential equations
2. Develop an Expected Results plot for your therapeutic agent concentration over time
3. Understand how initial conditions (C0 and t0) can be described in a diffusion problem (prep for COMSOL)

Additional Notes/Resources:
In order to run the MATLAB tutorial in newer versions of MATLAB, you will need to:

- Download datasets from Canvas
  - WAV_eeg_3_01_ORTH_CONT_1024.mat
  - WAV_eeg_3_01_ORTH_NONE_1024.mat
  - flu.mat
- Place datasets in your path (ie. folder that MATLAB is looking for files).

Grading Breakdown:
Q1: 8 points

1. Matlab code in .m file (3pts)
2. Matlab figure, within pdf document (3pts)
3. Conclusion statement, within pdf document (2pts)

Q2: 8 points

4. Project-to-code prep, within pdf document (2pts)
5. Matlab code in .m file (3pts)
6. Matlab figure, within pdf document (2pts)
7. Figure caption, created in pdf document (1pts)

Q3: 14 points

8. Matlab code in .m file (3pts)
9. Surface and Line plots of each of the three factors, within pdf document (9pts)
10. Written summary of findings, within pdf document (2pt)

Q1: Projectile Motion Tutorial Follow-up: Coffee cooling

Dr. Mays made a cup of coffee before class, however they didn't finish drinking it before class started. Use **Newton's Law of cooling** to model the convective heat exchange from the insulated mug to the 1620 BBBB classroom.

Newton's Law of cooling has an interesting history, that you can learn more about here, but for today, we will be using a very simplified version, which takes some calculus and ignores surface area (something you will learn more about in COMSOL 1). The simplified version is:

$$T(t) = (T_0 - T_a)\, e^{-ht} + T_a \qquad \textbf{(Eq. 1)}$$

Where:

- T(t) = Temperature, in fahrenheit (℉) with respect to time in minutes
- T_0 = Initial temperature of the coffee, ℉
- T_a = Ambient temperature of the room, ℉
- h = convective heat transfer constant, 1/minutes

After the 2-hour lecture period, what temperature will the coffee be? Will it still be "warm"?

Consider the following assumptions to be true:

- At the beginning of class, the coffee temperature is 190 ℉
- The ambient temperature of 1620 BBBB is 65 ℉
- h for a name brand, vacuum insulated tumbler, with no lid, is 0.014 (experimentally determined by Dr. Mays with a full cup of coffee)

For assignment submission:

1) Once complete, show your MATLAB code **(3pts)**, be sure to include:
    a) Your defined variables
    b) Your y = T(t) function
    c) Time increments for x, when temperature will be calculated every 1 minute for 120 minutes
    d) Updated comments to reflect your code's actions (as opposed to the tutorials's code)

2) Once complete, show your plot of coffee temperature over 2 hours **(3pts)**, with
    a) Descriptive title and axis labels with units
    b) x -axis includes time from 0 - 120 minutes
    c) y- axis is large enough to show all values of T(t) over that time frame
    d) A line representing the coffee temperature over time

3) By the end of class, is the coffee at least 115 ℉ or higher? If not, at what time, in minutes, during the lecture should Dr. Mays take a break to drink the rest of the coffee before it cools below 115 ℉? **(2pts)**

References for Newton's law of cooling equations:
*Besson, U. The History of the Cooling Law: When the Search for Simplicity can be an Obstacle. Sci & Educ **21**, 1085–1110 (2012).* https://doi-org.proxy.lib.umich.edu/10.1007/s11191-010-9324-1
J. Lienhard, J. Lienhard, *A Heat Transfer Textbook*. Phlogiston Press, 2020. Version 5.10, 14 August 2020, 784 pp, 28 MB, 8.5×11 in. (216 x 280 mm) https://ahtt.mit.edu/

Q2: Intro to Plotting follow-up: Expected Results Graph

Consider your Design Requirements related to safe and effective doses, and where you would measure such concentrations. Once in COMSOL, we will be calculating over specific volumes and measuring in specific places over those volumes, but for for the purposes of this question, we will assume that you have coated a flat surface with your therapeutic agent and that you are measuring concentration at a point (small volume) very close to that surface; i.e. our location of interest (LOI). We will also assume that the flat coated surface is in an aqueous environment, such that the therapeutic agent flows away and concentration at the LOI can be modeled with a natural decay function such as:

$$C(t) = C_0 e^{-kt} \qquad \textbf{(Eq. 2)}$$

Where

- C(t) = Concentration at time, t,
- C_0 = Initial concentration at LOI
- k = elimination rate constant
    - You could find something in literature for your drug of choice, or you can pick arbitrarily.
      If you pick arbitrarily:
        - Consider your data to be "mock" data that is only to be used temporarily
        - Suggestion: use a number between 0.2 and 1 (1/unit time)

  (Note how similar this equation is to Newton's Law of Cooling. This equation can also be used for pharmacokinetics or as plasma concentration over time after a single intravenous bolus dose)

Create a plot representing your expected results for your therapeutic agent concentration over time at the LOI.

4) To prepare yourself for creating your plot, write out answers to the following **(2pts)**:
    a) What therapeutic agent are you using?
    b) What is the upper limit for safe use? (specify your concentration units)
    c) What is the lower limit for effective use? (specify your concentration units)
    d) What C_0 do you plan to use?
        i) Suggestion: use a number close to, but not above, your upper limit for safety
    e) What timeframe should your therapeutic agent be present for? Days? Months? Years?
        i) Remember what was discussed in your User Needs drafts
        ii) Example: if your growth factor should be present for 56 weeks at a minimum to address the problem in 90% of the target population, then consider
            (1) t = 0:td:56, where td is 1 week
            (2) k = 0.3 in units of 1/week, picked arbitrarily (thus using dummy data)

5) For your MATLAB code **(3pts)**, be sure to include:
    a) Your defined variables
    b) Your y = C(t) function
    c) Time increments for x, using a time frame relevant to your project
    d) Updated comments to reflect your code's actions (as opposed to the tutorials's code)

6) Once complete, show your MATLAB plot for your expected results at your LOI and include **(2pts)**:
    a) A descriptive yet concise title
    b) Labeled x and y axes**, with units**
    c) A line representing concentration over time, using Eq 2
    d) Horizontal lines, using a high contrast color, to communicate your upper and lower concentration limits, as per your design requirements.
        i) Consider adding a legend to describe your horizontal lines if they would be difficult to describe in the caption

7) Write an appropriate caption to your figure including **(1pts)**:
    a) whether you are using mock data or not
    b) What the horizontal lines represent (give numbers, too)
    c) If the design requirement is met in this particular plot
        i) NOTE: it is not expected that you pass, we want you to **accurately label the data (mock or not) as pass / not pass** based on how your plot turned out

Q3: PDEPE tutorial follow-up: Pro-angiogenic growth factors transport in tissue engineered heart patch

In patients that have experienced a heart attack, heart muscle regeneration could one day be achieved through a tissue engineered heart patch. One **very important design constraint** in the development of such a cardiac patch **is the issue of mass transport**. Oxygen and nutrients need to diffuse from capillaries to cells within tissues to sustain cell and tissue viability. Additionally, in the case of angiogenesis, diffusive transport of factors (F) from cells in hypoxic (low oxygen) environments to existing capillary vessels is required to initiate vessel sprouting. Both transport processes depend on the physico-chemical properties of the tissue.

In order to model these diffusive transport processes experimentally, researchers have devised a 3D cell culture model of blood vessel growth **(Figure 1)**. In this model, the fibroblasts act as a rich source for pro-angiogenic molecules, which must diffuse through the gel to reach the endothelial cells.

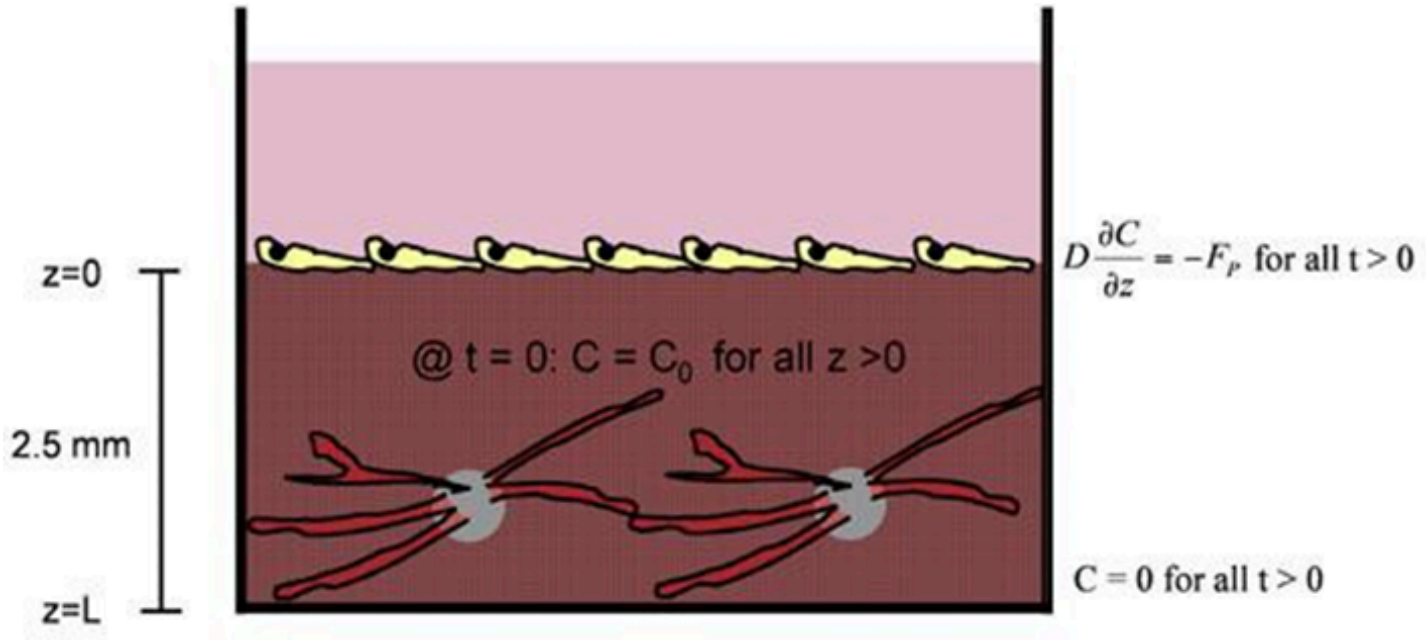

Figure 1. In this model system, endothelial cells(in red) are seeded on microcarrier beads (gray), embedded within a 3D gel (brown), and cultured underneath a monolayer of fibroblasts seeded on top of the gel (pale yellow). After a period of time, the endothelial cells coated on the beads undergo a process analogous to angiogenesis *in vivo.* The line of fibroblasts indicates where z = 0 and the endothelial cells are assumed to be concentrated at z = L = 2.5mm. Boundary conditions for z = 0 and z = L are listed to the right of the figure.

This experimental system has been used to ascertain how changes in the system (i.e., the separation between the fibroblasts and the endothelial cells, and the pore/mesh size of the matrix) affect the diffusion of fibroblast-secreted factors to govern angiogenesis. Mathematically, this system can be modeled as a 1-D, semi-infinite slab with the following boundary and initial conditions (BC and IC):

BC at z = 0: $D * \partial C/\partial z = - F_P$ for all of t>0

BC at z = L: C = 0 for all t >0

IC at t = 0: $C = C_0$ for all z

These conditions simulate a soluble macromolecule (e.g., a protein or **F**actor), initially at a concentration C_0, produced at a constant rate, F_P, at the gel surface (z = 0) by fibroblasts, and immediately consumed when it reaches the endothelial cells by diffusion at the bottom of the gel (a distance L away).

Additional Provided values:

$C_0 = 0$, the initial concentration of the fibroblast-derived factor is 0, for all three factors

$F_P = 4.94 \times 10^{-19}$ mol/cm^2•s , the rate at which the fibroblasts make the factor of interest

$D_1 = -10^{-6}$ cm^2/sec, diffusion coefficient for factor 1, F_1

$D_2 = -10^{-7}$ cm^2/sec, diffusion coefficient for factor 2, F_2

$D_3 = -10^{-8}$ cm^2/sec, diffusion coefficient for factor 3, F_3

8) Develop the Matlab program that yields a plot of concentration as a function of distance (from z=0 to z=L) and time (from t=0 until a steady state is reached). Do this for each of the three factors F_1, F_2, and F_3 **(3pts)** Hints and Tips:
    a) Use the PDEPE function in MATLAB to solve these
    b) Note/Reminder: You are working in two dimensions, distance (z-direction) and time
    c) Steady state will be achieved when the concentration over distance no longer changes as time progresses
        i) You may need to iterate until you find no more changes (line plots might help, see 9)
9) For each Factor, create a plot that includes 2 subplots **(9pts):**
    a) Surface plot showing concentration over distance and time
    b) Line plot showing concentration over distance, at the time steady state is achieved.
        i) Indicate the time in the title of the subplot
    c) Include descriptive titles and axis labels for all plots
10) Briefly summarize your findings from this task **(2pt)**