MAT 215 ()

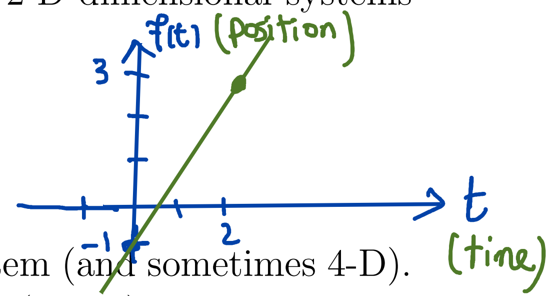
Chapter 12

1 Section 12.1: Coordinate Systems

- In previous calculus classes (calc 1 and 2 for example), you studied functions of one variable ($f(x)$) and you worked in 1- D and 2-D dimensional systems (like $\mathbb{R}$ and $\mathbb{R}^2$).

Example: $f(t)$: The position of a particle at time t.

$f(t) = 2t - 1$



- In this course, we will work in a 3-D coordinate system (and sometimes 4-D). We will deal with multivariable functions: $f(x, y)$, $f(x, y, z)$,...

- **Examples:**

Area of a rectangle: $A(L,w) = L \cdot w$.

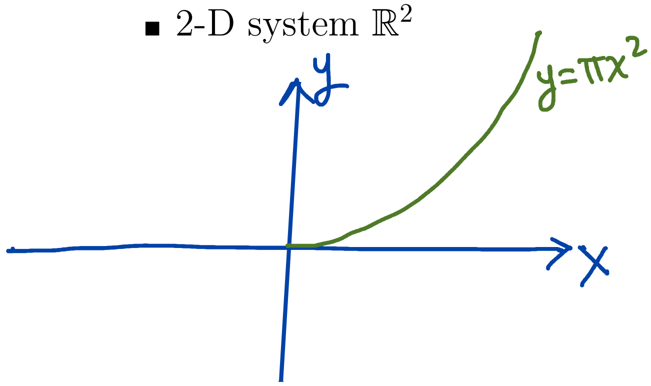
Volume of a cylinder $V(r,h) = \pi r^2 h$

Area of a rectangular box: $A(L,w,h) = L \cdot w \cdot h$.

- **The standard (rectangular) coordinate systems**
  - 1-D system $\mathbb{R}$

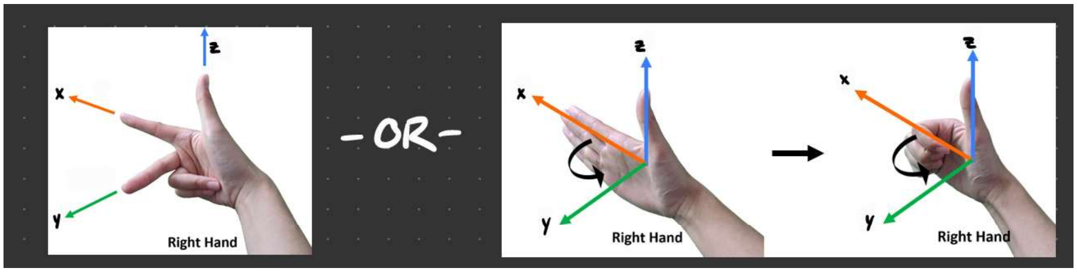
Can be used to represent the position of a particle moving on a straight line

  - 2-D system $\mathbb{R}^2$



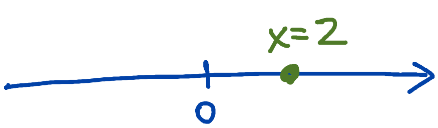
Area of a circle: $A(x) = \pi x^2$

- 3-D system $\mathbb{R}^3$ (Determined by the right hand rule)

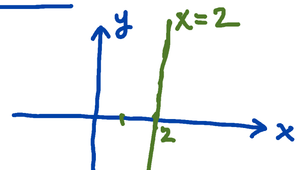


- **Examples:**
  - What type of geometric object does $x = 2$ represent in each of $\mathbb{R}$, $\mathbb{R}^2$, and $\mathbb{R}^3$?

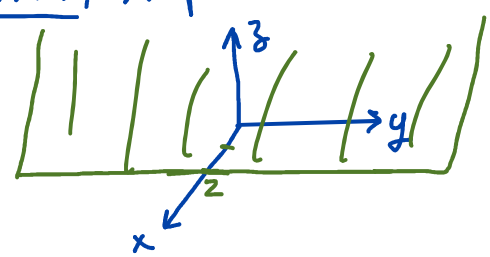
In $\mathbb{R}$: A point



In $\mathbb{R}^2$: A line

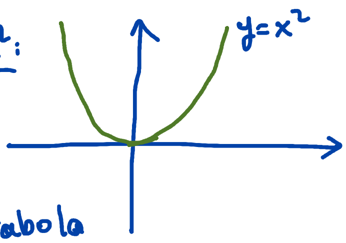


In $\mathbb{R}^3$; A plane



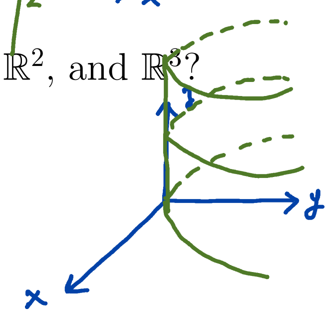
  - How about $y = x^2$? in each of $\mathbb{R}^2$, and $\mathbb{R}^3$?

In $\mathbb{R}^2$:



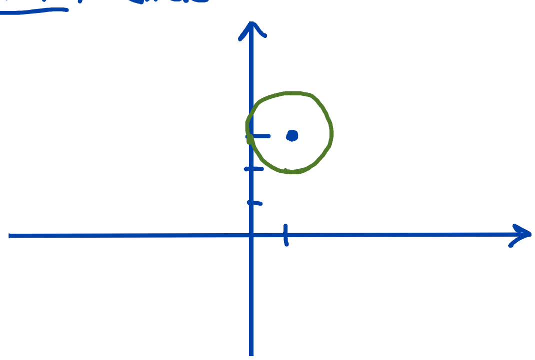
Parabola

In $\mathbb{R}^3$:

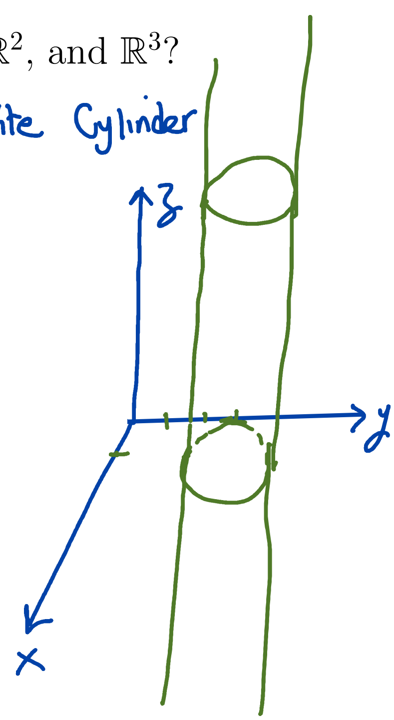


  - How about $(x-1)^2 + (y-3)^2 = 1$? in each of $\mathbb{R}^2$, and $\mathbb{R}^3$?

In $\mathbb{R}^2$; Circle

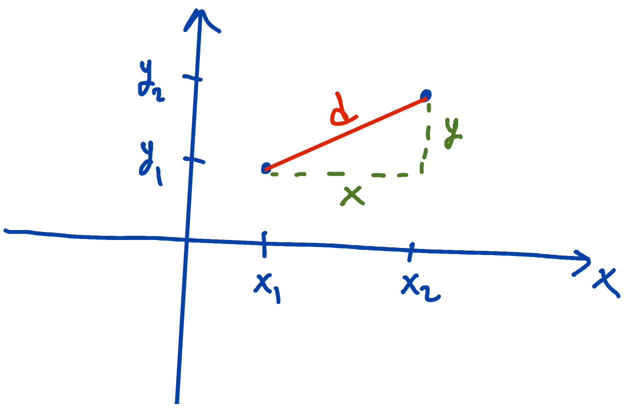


In $\mathbb{R}^3$; Infinite Cylinder



- **Distance Formulas**
  - **Distance formula in $\mathbb{R}^2$:** The distance between the points $p_1 = (x_1, y_1)$ and $p_2 = (x_2, y_2)$ in $\mathbb{R}^2$ is equal to

$$d(p_1, p_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$



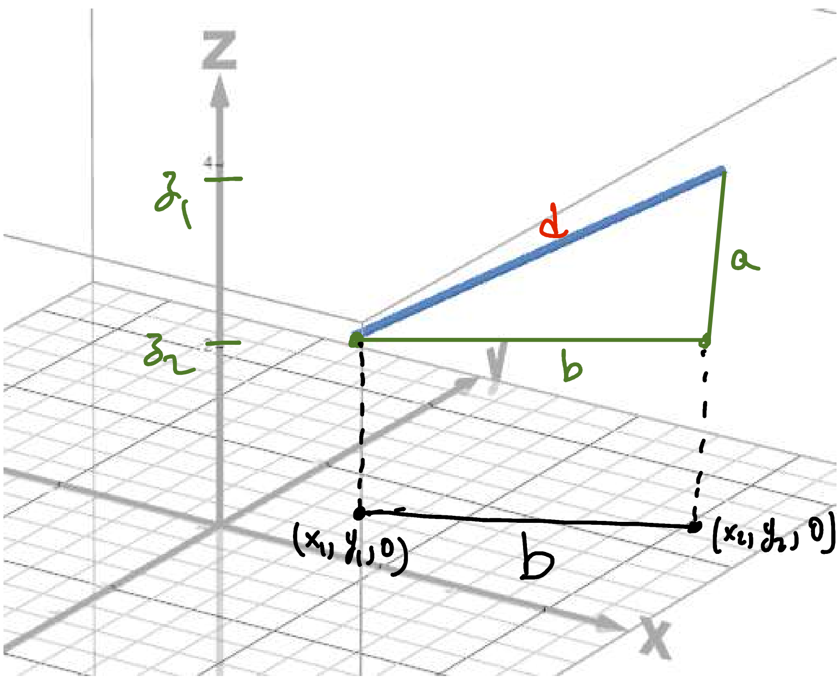
$d^2 = x^2 + y^2$

$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

so, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

  - **Distance formula in $\mathbb{R}^3$:** The distance between the points $p_1 = (x_1, y_1, z_1)$ and $p_2(x_2, y_2, z_2)$ in $\mathbb{R}^3$ is equal to

$$d(p_1, p_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$



$d^2 = a^2 + b^2$

where

$a^2 = (z_2 - z_1)^2$ and

$b^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

Then,

$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$

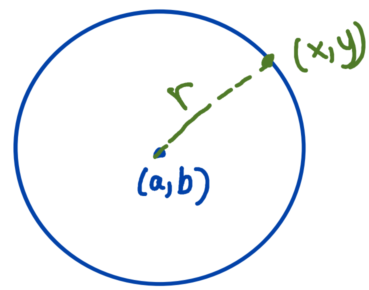
- Examples

1. Find an equation of the circle in $\mathbb{R}^2$ of radius r and center (a, b).

$P=(x,y)$ is on the circle if the distance between P and the center is equal to r.

That is, $\sqrt{(x-a)^2+(y-b)^2}=r$

$$(x-a)^2+(y-b)^2=r^2$$



2. Find an equation of the sphere in $\mathbb{R}^3$ of radius r and center (a, b, c).

A point $P(x,y,z)$ is on the sphere if the distance between P and (a,b,c) is equal to r

$$\sqrt{(x-a)^2+(y-b)^2+(z-c)^2}=r$$

$$(x-a)^2+(y-b)^2+(z-c)^2=r^2$$

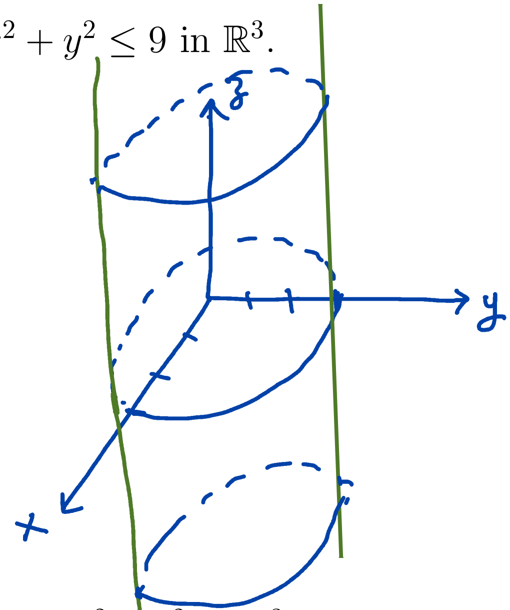
3. Sketch the surface $x^2 + y^2 = 4$ in $\mathbb{R}^3$.

(We did something similar above)

4. Describe the region $x^2 + y^2 \leq 9$ in $\mathbb{R}^3$.

solid infinite Cylinder

of radius 3



5. Sketch the surface of $z = x^2 + y^2$ in $\mathbb{R}^3$ *(it is called a circular paraboloid)*.

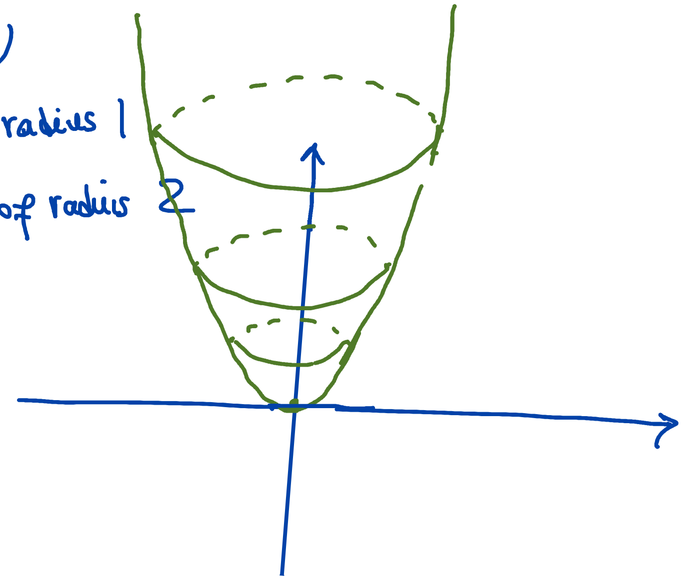
$z = x^2+y^2$.

For $z=0$, $x^2+y^2=0$, $(x,y) = (0,0)$

For $z=1$, $x^2+y^2=1$ Circle of radius 1

For $z=4$ $x^2+y^2=2$ Circle of radius 2

⋮



6. Describe the region $x^2 + y^2 + z^2 \leq 25$.

Solid ball.

A point $p(x,y,z)$ is in the region if $d(P, \text{origin}) \leq 5$.