4 Section 12.4: The Cross Product

- **Definition:** Let $\vec{u} = (a, b, c)$ and $\vec{v} = (d, e, f)$ be two vectors. The **cross product** $\vec{u} \times \vec{v}$ is a **vector** defined by

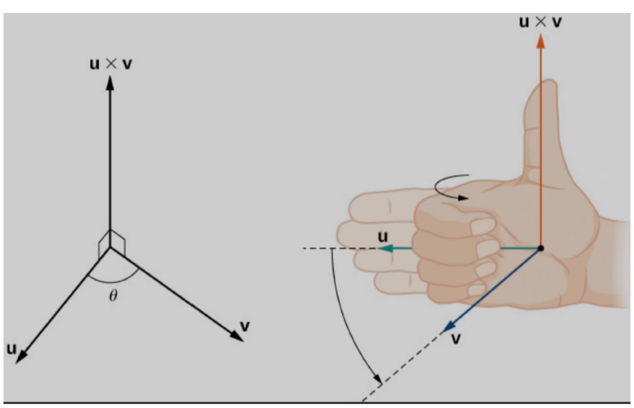
$$\vec{u} \times \vec{v} = \langle bf - ce, -(af - cd), ae - bd \rangle$$

.

  - In matrix determinant notation:

  - **Note:** *There is no cross product in 2-D. But if you wish to find the cross product of two vectors in* $\mathbb{R}^2$*, then you can assume that the* z *component is zero.*

- **Direction of the cross product vector: Right Hand Rule**



- **Algebraic properties of the cross product**
  - $\vec{u} \times \vec{u} = \vec{0}$
  - $\vec{u} \times \vec{v} = -\vec{v} \times \vec{u}$
  - $\vec{u} \times (\vec{v} + \vec{w}) = \vec{u} \times \vec{v} + \vec{u} \times \vec{w}$
  - $(r\vec{u}) \times \vec{v} = r(\vec{u} \times \vec{v})$
  - $\vec{u} \times \vec{0} = \vec{0}$

What does the cross product say about the relationship between two given vectors?

- **Theorem: The cross product $\vec{u} \times \vec{v}$ is orthogonal to both of the vectors $\vec{u}$ and $\vec{v}$**

  Why? *try finding a dot product...*

- **Theorem:** Given two vectors $\vec{u}$ and $\vec{v}$, Let θ be the angle between them $(0 \leq \theta \leq \pi)$. Then

$$\|\vec{u} \times \vec{v}\| = \|\vec{u}\|\|\vec{v}\| \sin(\theta)$$

  and

  How can we prove this?

  Hint: by direct computation, you can find: $\|\vec{u} \times \vec{v}\|^2 = \|\vec{u}\|^2\|\vec{v}\|^2 - (\vec{u} \cdot \vec{v})^2$, *then continue from there...*

- **Corollary:** Two non-zero vectors $\vec{u}$ and $\vec{v}$ are parallel if and only if $\vec{u} \times \vec{v} = 0$

Interpretation:

- **Example:** Find two unit vectors which are orthogonal to $\vec{u} = (-1, 0, 3)$.

- **Finding the area of a parallelogram using cross product.**