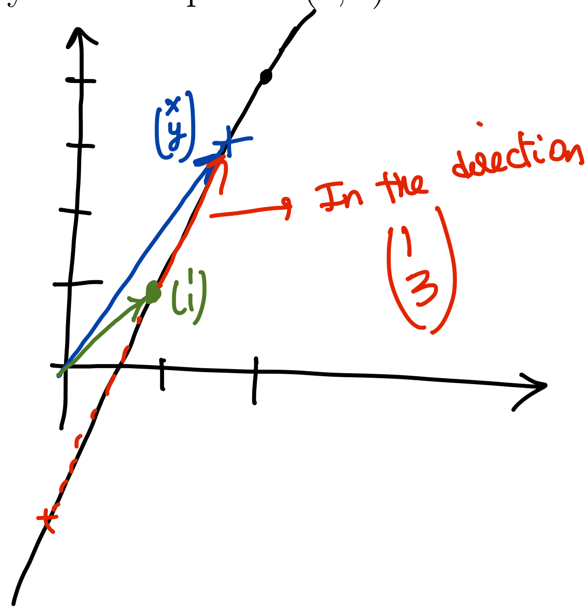
5 Section 12.5: Equations of lines and planes

- In $\mathbb{R}^2$, how do we describe a line that passes by the two points $(1,1)$ and $(2,4)$?



  - Relation between y and x:

slope $= \frac{3}{1} = 3$ $\quad y - 1 = 3(x-1)$

$\boxed{y = 3x - 2}$

  - Vector notation:

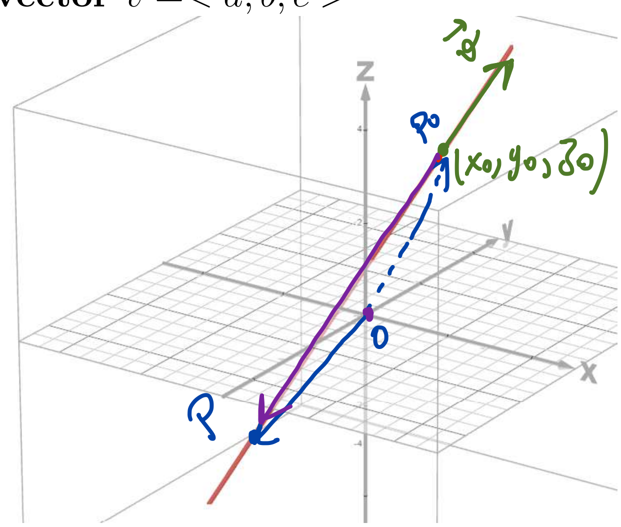
$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} t$ or $\langle x, y \rangle = \langle 1, 1 \rangle + \langle 1, 3 \rangle t$

  - Parametric equations:

$x(t) = 1 + t, \quad y(t) = 1 + 3t$

or $\quad L(t) = \langle 1+t, 1+3t \rangle$

- **Equations of a line in $\mathbb{R}^3$ that passes by a point $P = (x_0, y_0, z_0)$ and parallel to the vector $\vec{v} = < a, b, c >$**



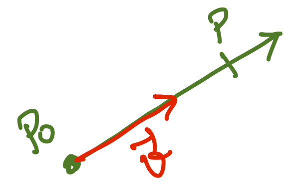
  - **Vector form:**

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + t\vec{v}$ so, $\vec{OP} = \vec{OP_0} + t\vec{v}$

- **Parametric form:**

$x_{(t)} = x_0 + ta, \quad y_{(t)} = y_0 + tb, \quad z_{(t)} = z_0 + tc$

or $\langle x_0 + ta, \ y_0 + tb, \ z_0 + tc \rangle$



- **Symmetric form:**

$t = \frac{x - x_0}{a}$ so, $\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$

$t = \frac{y - y_0}{b}$ what does this say? Note that $\begin{pmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{pmatrix} = \vec{PP_0}$

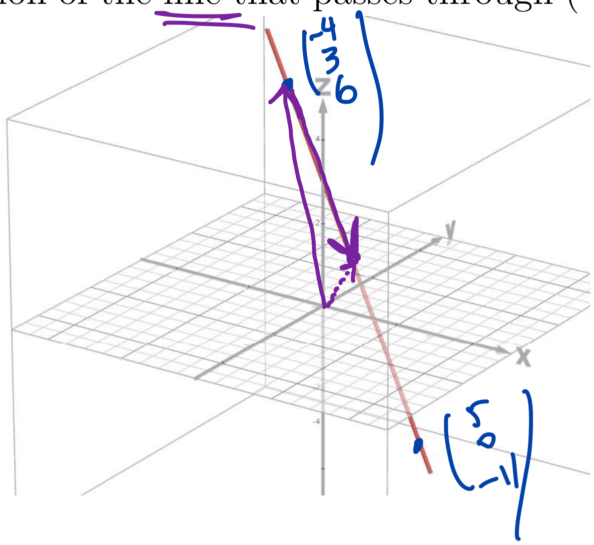
$t = \frac{z - z_0}{c}$

so, $\vec{P_0P}$ and $\vec{v}$ are parallel.

- **Examples:**

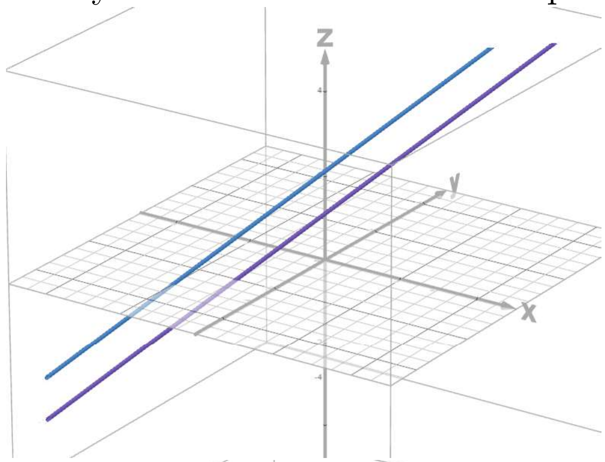
  - Find an equation of the line that passes through $(-4, 3, 6)$ and $(1, 3, -5)$.

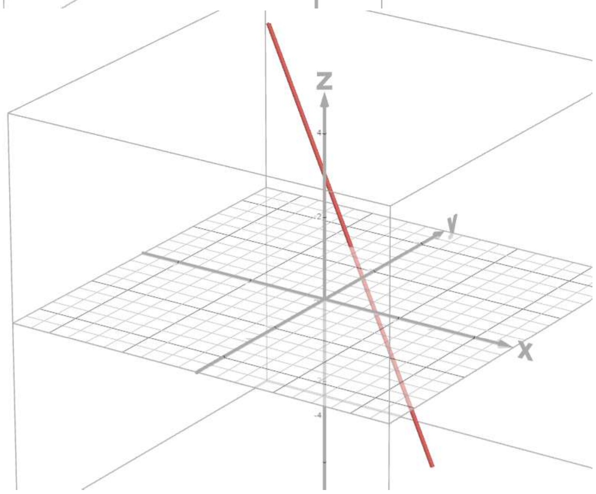
$\begin{pmatrix} -4 \\ 3 \\ 6 \end{pmatrix} + t \begin{pmatrix} 5 \\ 0 \\ -11 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ ✓



  - Find an equation of the line segment between $(-4, 3, 6)$ and $(1, 3, -5)$.

- What would you say about two lines that have parallel direction vectors?





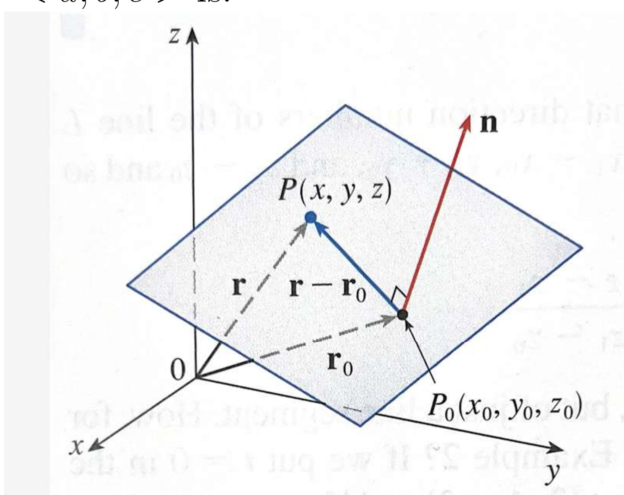
  * In the case that the two lines have a common point

  * In the case that the two lines do not have any common point

- **Def:** Given a plane, a **normal vector** is a vector orthogonal to every line in the plane.

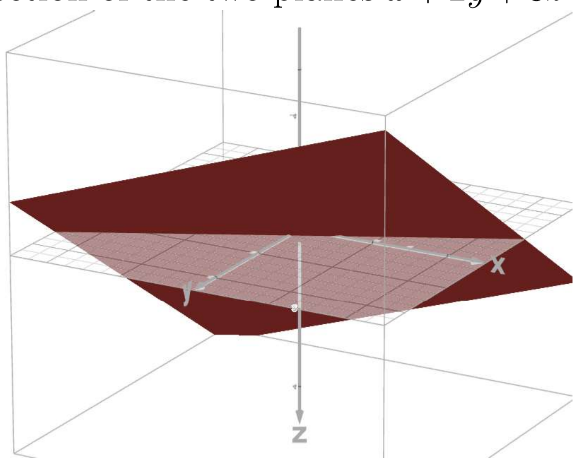
- **Examples:**
  - Find normal vectors for the xy-plane, xz-plane, and yz-plane.
  - Describe the set of all the vectors orthogonal to the vector $< -1, 3, 2 >$.

- The equation of a **plane** which contains the point $p = (x_0, y_0, z_0)$ and has a norma vector $\vec{n} =< a, b, c >$ is:



  - **Vector form:**
  - **Scalar form:**
  - **General form:**

- **Examples:**
  - Find an equation of the plan though $(0, 1, 1)$, $(1, 0, 1)$ and $(1, 1, 0)$.
  - Find an equation for the line perpendicular to $3x - 2y + 2z = 8$ and which passes through $(5, 7, 1)$.
  - Find the point of intersection of the line $L(t) = (2 - 2t, 3t, 1 + t)$ and the plane $x + 2y - z = 7$.
  - Find the intersection of the two planes $x + 2y + 3z = 1$ and $x - y + z = 1$.



  - Find the angle between the two planes $x + 2y - z = 2$ and $2x - 2y + z = 1$.
  - Find the distance between the point $(4, 1, -2)$ and the line $L(t) = (1 + t, 3 - 2t, 4 - 3t)$.
  - Find the distance between the point $(1, -2, 4)$ and the plane $3x + 2y + 6z = 5$.
  - Find the distance between the two lines with parametric equations $L(t) = (1 + t, 1 + 6t, 2t)$ and $M(t) = (1 + 2s, 5 + 15s, -2 + 6s)$.
  - Do the lines $L = (t, -t, t)$ and $M = (2t + 3, t, t + 2)$ intersect? If so, what is their intersection?